

БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
МЕХАНИКО-МАТЕМАТИЧЕСКИЙ ФАКУЛЬТЕТ
Кафедра теории функций

МАТЕМАТИЧЕСКИЙ АНАЛИЗ

ИНДИВИДУАЛЬНЫЕ ЗАДАНИЯ
для студентов механико-математического факультета

МИНСК
БГУ
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ТРЕБОВАНИЯ К ОФОРМЛЕНИЮ ИНДИВИДУАЛЬНЫХ ЗАДАНИЙ

Студент выполняет индивидуальные задания в отдельной тетради, оставляя поля для замечаний преподавателя.

На обложке тетради студент указывает свою фамилию, имя, номер учебной группы и вариант индивидуального задания.

Решения задач следует излагать в порядке номеров, указанных в задании.

Решения задач излагать **подробно и аккуратно, выполняя все необходимые теоретические обоснования.**

ИНДИВИДУАЛЬНОЕ ЗАДАНИЕ 9 «ОПРЕДЕЛЕННЫЙ ИНТЕГРАЛ»

1. Вычислить по определению:

1. $\int_2^3 x^2 dx.$

2. $\int_1^2 2^x dx.$

3. $\int_1^3 3^x dx.$

4. $\int_1^2 4^x dx.$

5. $\int_{\pi/4}^{\pi/2} \cos x dx.$

6. $\int_{\pi/4}^{\pi/2} \sin x dx.$

7. $\int_1^2 (x^2 - x) dx.$

8. $\int_1^3 x^3 dx.$

9. $\int_{\pi/3}^{\pi/2} \sin x dx.$

10. $\int_{-1}^2 3^x dx.$

11. $\int_{\pi/2}^{\pi} \cos x dx.$

12. $\int_1^2 (x^2 + x) dx.$

13. $\int_{-1}^2 2^x dx.$

14. $\int_{\pi/3}^{\pi/2} \cos x dx.$

15. $\int_{-1}^2 e^x dx.$

16. $\int_1^3 (2x^2 - 3x + 1) dx.$

17. $\int_3^4 x^2 dx.$

18. $\int_{\pi/2}^{\pi} \sin x dx.$

19. $\int_{-1}^2 4^x dx.$

20. $\int_1^4 (3x^2 - 4x + 1) dx.$

21. $\int_3^5 x^2 dx.$

22. $\int_0^{\pi/2} \sin x dx.$

2. С помощью определенных интегралов найти пределы следующих сумм:

1. $\lim_{n \rightarrow \infty} n^2 \sum_{k=1}^n \frac{1}{n^3 + k^3}.$

2. $\lim_{n \rightarrow \infty} n \sum_{k=1}^n \frac{1}{n^2 + k^2}.$

3. $\lim_{n \rightarrow \infty} \sin \frac{\pi}{n} \sum_{k=1}^n \sin \frac{k\pi}{n}.$

4. $\lim_{n \rightarrow \infty} \sin \frac{\pi}{n} \sum_{k=1}^n \cos \frac{k\pi}{n}.$

$$5. \lim_{n \rightarrow \infty} \operatorname{tg} \frac{1}{n} \sum_{k=1}^n \operatorname{tg} \left(-\frac{\pi}{4} + \frac{k\pi}{2n} \right).$$

$$6. \lim_{n \rightarrow \infty} \sin \frac{1}{n} \sum_{k=1}^n \sqrt{1 + \frac{k}{n}}.$$

$$7. \lim_{n \rightarrow \infty} \ln \left(1 + \frac{1}{n} \right) \sum_{k=1}^n \sqrt{1 + \frac{k^2}{n^2}}.$$

$$8. \lim_{n \rightarrow \infty} \operatorname{tg} \frac{1}{n} \sum_{k=1}^n \frac{k}{n} e^{k/n}.$$

$$9. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^2}{n^3} \sin \frac{k^3}{n^3}.$$

$$10. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{5n^2 - k^2}}.$$

$$11. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\sqrt{k+n}}{n^{3/2}}.$$

$$12. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{2n+k}.$$

$$13. \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{k}{\sqrt{k^2 + n^2}}.$$

$$14. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{5n^2 + k^2}}.$$

$$15. \lim_{n \rightarrow \infty} \ln \left(1 - \frac{3}{n^2} \right) \sum_{k=1}^n \frac{k}{1 + \left(\frac{k}{n} \right)^2}.$$

$$16. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{k^2 + n^2}.$$

$$17. \lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{k=1}^n \sqrt{n^2 + k^2}.$$

$$18. \lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{k=1}^n 2^{k/n} k.$$

$$19. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^3}{n^4} \cos \frac{k^4}{n^4}.$$

$$20. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{4n^2 - 3k^2}}.$$

$$21. \lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{k=1}^n \sqrt{k^2 + n^2}.$$

$$22. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{3n+2k}.$$

3. Вычислить интегралы:

1.

$$a) \int_0^{1/\sqrt{3}} \frac{dx}{(2x^2+1)\sqrt{x^2+1}};$$

$$б) \int_0^1 x \arccos x \, dx;$$

$$B) \int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{x^2-1}};$$

$$Г) \int_0^1 \frac{x^3+2x^2+4x+1}{x^3+x^2+x+1} dx;$$

$$Д) \int_{-\pi/3}^{\pi/3} \left(x^2 \sin 5x + \cos \frac{x}{3} + \operatorname{tg}^3 x \right) dx; \quad e) \int_0^{\pi} e^x \cos^2 x \, dx;$$

$$\text{Ж)} \int_{-\pi/2}^{3\pi/2} \left| \cos^3 \frac{x}{2} \right| dx;$$

$$\text{И)} \int_{-2}^0 (x^2 + 5x + 6) \cos 2x dx;$$

$$\text{Л)} \int_{\pi/2}^{2 \operatorname{arctg} 2} \frac{dx}{\sin^2 x (1 - \cos x)};$$

$$\text{Н)} \int_{\pi/2}^{\pi} 2^8 \sin^8 x dx;$$

$$\text{П)} \int_0^{16} \sqrt{256 - x^2} dx.$$

$$\text{З)} \int_{-1/2}^3 (|x| + 1)[x] dx;$$

$$\text{К)} \int_{e+1}^{e^2+1} \frac{1 + \ln(x-1)}{x-1} dx;$$

$$\text{М)} \int_{\pi/4}^{\operatorname{arctg} 3} \frac{dx}{(3 \operatorname{tg} x + 5) \sin 2x};$$

$$\text{О)} \int_0^1 \frac{4\sqrt{1-x} - \sqrt{3x+1}}{(\sqrt{3x+1} + 4\sqrt{1-x})(3x+1)^2} dx;$$

2.

$$\text{а)} \int_0^4 \sqrt{4x - x^2} dx;$$

$$\text{В)} \int_{2/\sqrt{3}}^2 \frac{dx}{x\sqrt{x^2-1}};$$

$$\text{Д)} \int_{-\pi/3}^{\pi/3} \frac{2x^7 - x^5 + 2x^3 - x + 1}{\cos^2 x} dx;$$

$$\text{Ж)} \int_{1/e}^e |x \ln x| dx;$$

$$\text{И)} \int_{-2}^0 (x^2 - 4) \cos 3x dx;$$

$$\text{Л)} \int_0^{\pi/2} \frac{\cos x dx}{2 + \cos x};$$

$$\text{Н)} \int_0^{\pi} 2^4 \sin^6 x \cos^2 x dx;$$

$$\text{П)} \int_0^1 x^2 \sqrt{1 - x^2} dx.$$

$$\text{б)} \int_0^{\pi} e^x \cos x dx;$$

$$\text{Г)} \int_0^1 \frac{dx}{\sqrt{x^2 + 2x + 2}};$$

$$\text{е)} \int_0^{\pi} e^x \sin^2 x dx;$$

$$\text{з)} \int_{-\pi/2}^{\pi/2} \cos^2 x \operatorname{sgn}(\sin x) dx;$$

$$\text{К)} \int_0^1 \frac{x^2 + 1}{(x^3 + 3x + 1)^2} dx;$$

$$\text{М)} \int_{\arccos \frac{4}{\sqrt{17}}}^{\pi/4} \frac{2 \operatorname{ctg} x + 1}{(2 \sin x + \cos x)^2} dx;$$

$$\text{О)} \int_1^{64} \frac{1 - \sqrt[6]{x} + 2\sqrt[3]{x}}{x + 2\sqrt{x^3} + \sqrt{x^4}} dx;$$

3.

а) $\int_0^2 x^2 \sqrt{4-x^2} dx;$

б) $\int_{2/\sqrt{3}}^{\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}};$

в) $\int_0^{2\pi} x^2 \cos x dx;$

г) $\int_{-1/2}^2 ||x| - 1| dx;$

д) $\int_{-1}^0 (x^2 + 4x + 3) \cos x dx;$

е) $\int_{\pi/2}^{2 \operatorname{arctg} 2} \frac{dx}{\sin^2 x (1 + \cos x)};$

ж) $\int_0^{2\pi} \sin^4 x \cos^4 x dx;$

з) $\int_0^5 \frac{dx}{(25+x^2)\sqrt{25+x^2}}.$

б) $\int_0^1 \arccos \sqrt{x} dx;$

г) $\int_{3/4}^1 \frac{dx}{\sqrt{2+3x-2x^2}};$

е) $\int_{2\pi}^{3\pi} e^x \cos^2 x dx;$

з) $\int_{1/e}^8 [\ln x] e^{3x} dx;$

к) $\int_0^1 \frac{4 \operatorname{arctg} \frac{x-x}{1+x^2}}{1+x^2} dx;$

м) $\int_0^{\arccos \frac{1}{\sqrt{17}}} \frac{3+2 \operatorname{tg} x}{2 \sin^2 x + 3 \cos^2 x - 1} dx;$

о) $\int_{-14/15}^{-7/8} \frac{6\sqrt{x+2} dx}{(x+2)^2 \sqrt{x+1}};$

4.

а) $\int_0^1 \frac{e^{2x} + 2e^x}{e^{2x} + 1} dx;$

б) $\int_2^3 \frac{dx}{x\sqrt{x^2-1}};$

в) $\int_0^{\ln 2} \sqrt{e^x - 1} dx;$

г) $\int_{1/e^2}^e \left| \ln \frac{x}{2} \right| dx;$

д) $\int_{-2}^0 (x+2)^2 \cos 3x dx;$

б) $\int_0^{\pi/2} \frac{dx}{2-\sin x};$

г) $\int_0^1 \frac{4x^3 + 8x^2 + 9x + 6}{4x^2 + 4x + 5} dx;$

е) $\int_{2\pi}^{3\pi} e^x \sin^2 x dx;$

з) $\int_{\pi/4}^{3\pi/4} \operatorname{sgn}(\cos x) \operatorname{ctg} x dx;$

к) $\int_0^2 \frac{x^3 dx}{x^2 + 4};$

$$\text{Л)} \int_{2 \operatorname{arctg} \frac{1}{2}}^{\pi/2} \frac{\cos x \, dx}{(1 - \cos x)^3};$$

$$\text{Н)} \int_0^{2\pi} \sin^2 \frac{x}{4} \cos^6 \frac{x}{4} \, dx;$$

$$\text{П)} \int_0^3 \frac{dx}{(9+x^2)^{3/2}}.$$

$$\text{М)} \int_{\pi/4}^{\operatorname{arctg} 3} \frac{4 \operatorname{tg} x - 5}{1 - \sin 2x + 4 \cos^2 x} \, dx;$$

$$\text{О)} \int_6^9 \sqrt{\frac{9-2x}{2x-21}} \, dx;$$

5.

$$\text{а)} \int_1^{\sqrt[3]{e}} \frac{dx}{x \sqrt{1 - \ln^2 x}};$$

$$\text{В)} \int_{\sqrt{2}}^4 \frac{dx}{x \sqrt{x^2 - 1}};$$

$$\text{Д)} \int_1^3 \operatorname{arctg} \sqrt{x} \, dx;$$

$$\text{Ж)} \int_{-3}^4 |x| e^{|x^2 - 4|} \, dx;$$

$$\text{И)} \int_{-4}^0 (x^2 + 7x + 12) \cos x \, dx;$$

$$\text{Л)} \int_0^{\pi/2} \frac{\cos x - \sin x}{(1 + \sin x)^2} \, dx;$$

$$\text{Н)} \int_0^{\pi} 2^4 \cos^8 \frac{x}{2} \, dx;$$

$$\text{П)} \int_0^{\sqrt{5}/2} \frac{dx}{\sqrt{(5-x^2)^3}}.$$

$$\text{б)} \int_0^{1/2} \frac{\arcsin \sqrt{x}}{\sqrt{x(1-x)}} \, dx;$$

$$\text{Г)} \int_0^4 \frac{dx}{1 + \sqrt{2x+1}};$$

$$\text{е)} \int_{2\pi}^{3\pi} e^{-x} \cos^2 x \, dx;$$

$$\text{з)} \int_1^4 \frac{\ln^2[x]}{x} \, dx;$$

$$\text{К)} \int_{\pi}^{2\pi} \frac{x + \cos x}{x^2 + 2 \sin x} \, dx;$$

$$\text{М)} \int_0^{\operatorname{arctg} \frac{1}{3}} \frac{8 + \operatorname{tg} x}{18 \sin^2 x + 2 \cos^2 x} \, dx;$$

$$\text{о)} \int_0^5 e^{\sqrt{\frac{5-x}{5+x}}} \frac{dx}{(5+x)\sqrt{25-x^2}};$$

6.

$$\text{а)} \int_0^a \sqrt{ax - x^2} \, dx;$$

$$\text{В)} \int_2^5 \frac{dx}{x \sqrt{x^2 - 1}};$$

$$\text{б)} \int_1^{16} \operatorname{arctg} \sqrt{\sqrt{x} - 1} \, dx;$$

$$\text{Г)} \int_4^9 \frac{\sqrt{x} \, dx}{\sqrt{x-1}};$$

$$д) \int_0^1 \arccos x \, dx;$$

$$ж) \int_0^1 e^{|2-3x|} dx;$$

$$и) \int_0^{\pi} (2x^2 + 4x + 7) \cos 2x \, dx;$$

$$л) \int_{2 \operatorname{arctg} 2}^{2 \operatorname{arctg} 3} \frac{dx}{\cos x(1-\cos x)};$$

$$н) \int_{-\pi/2}^0 2^8 \sin^8 x \, dx;$$

$$п) \int_1^2 \frac{\sqrt{x^2-1}}{x^4} dx.$$

$$е) \int_{2\pi}^{3\pi} e^{-x} \sin^2 x \, dx;$$

$$з) \int_{-1}^3 x \operatorname{sgn}(x^5 - 4x^4) dx;$$

$$к) \int_0^{\pi/4} \frac{2 \cos x + 3 \sin x}{(2 \sin x - 3 \cos x)^3} dx;$$

$$м) \int_0^{\arccos \sqrt{2/3}} \frac{\operatorname{tg} x + 2}{\sin^2 x + 2 \cos^2 x - 3} dx;$$

$$о) \int_8^{12} \sqrt{\frac{6-x}{x-14}} dx;$$

7.

$$а) \int_0^1 x^3 \sqrt{x^2 + 9} \, dx;$$

$$в) \int_{\frac{3}{4}}^{\frac{4}{3}} \frac{dx}{x \sqrt{x^2-1}};$$

$$д) \int_0^1 \arcsin \sqrt{x} \, dx;$$

$$ж) \int_{-1}^1 \sqrt{|(2x-1)^3|} dx;$$

$$и) \int_0^{\pi} (9x^2 + 9x + 11) \cos 3x \, dx;$$

$$л) \int_{2 \operatorname{arctg} \frac{1}{3}}^{2 \operatorname{arctg} \frac{1}{2}} \frac{dx}{\sin x(1-\sin x)};$$

$$н) \int_{\pi/2}^{\pi} 2^4 \sin^6 x \cos^2 x \, dx;$$

$$б) \int_0^3 \ln(x+3) dx;$$

$$г) \int_1^2 \frac{e^{1/x}}{x^3} dx;$$

$$е) \int_0^{\pi} e^{2x} \cos^2 2x \, dx;$$

$$з) \int_{-1}^2 x^3 [e^{-x}] dx;$$

$$к) \int_0^{1/2} \frac{8x - \operatorname{arctg} 2x}{1+4x^2} dx;$$

$$м) \int_{\arcsin \frac{1}{\sqrt{37}}}^{\pi/4} \frac{6 \operatorname{tg} x}{3 \sin 2x + 5 \cos^2 x} dx;$$

$$о) \int_0^1 e^{\sqrt{\frac{1-x}{1+x}}} \frac{dx}{(1+x)\sqrt{1-x^2}};$$

$$\text{II)} \int_0^{\sqrt{2}/2} \frac{x^4 dx}{\sqrt{(1-x^2)^3}}.$$

8.

$$\text{a)} \int_{\ln 2}^{\ln 6} \frac{e^x \sqrt{e^x - 2}}{e^x + 2} dx;$$

$$\text{б)} \int_0^1 x^2 \operatorname{arctg} x dx;$$

$$\text{B)} \int_{1/4}^{1/5} \frac{dx}{x\sqrt{1-x^2}};$$

$$\text{Г)} \int_1^e \frac{\cos(\ln x)}{x} dx;$$

$$\text{Д)} \int_0^1 x^2 \sqrt{1-x^2} dx;$$

$$\text{e)} \int_0^{\pi} e^{2x} \sin^2 2x dx;$$

$$\text{Ж)} \int_{-e/3}^{-1/3} |\ln |3x|| dx;$$

$$\text{з)} \int_0^5 \frac{\operatorname{sgn}(x^2 - 5x + 6)}{x^2 + 1} dx;$$

$$\text{И)} \int_0^{\pi} (8x^2 + 16x + 17) \cos 4x dx;$$

$$\text{К)} \int_1^4 \frac{\frac{1}{2\sqrt{x}} + 1}{(\sqrt{x} + x)^2} dx;$$

$$\text{Л)} \int_{2 \operatorname{arctg} \frac{1}{2}}^{\pi/2} \frac{dx}{(1 + \sin x - \cos x)^2};$$

$$\text{M)} \int_0^{\pi/4} \frac{2 \operatorname{tg}^2 x - 11 \operatorname{tg} x - 22}{4 - \operatorname{tg} x} dx;$$

$$\text{H)} \int_0^{\pi} 2^4 \sin^4 x \cos^4 x dx;$$

$$\text{O)} \int_{5/2}^{10/3} \frac{\sqrt{x+2} + \sqrt{x-2}}{(\sqrt{x+2} - \sqrt{x-2})(x-2)^2} dx;$$

$$\text{II)} \int_0^{\sqrt{3}} \frac{dx}{\sqrt{(4-x^2)^3}}.$$

9.

$$\text{a)} \int_{\sqrt{8/3}}^{2\sqrt{2}} \frac{dx}{x\sqrt{(x^2-2)^5}};$$

$$\text{б)} \int_0^{\pi/2} e^x \sin x dx;$$

$$\text{B)} \int_{-2}^{-\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}};$$

$$\text{Г)} \int_0^1 \frac{dx}{e^x + e^{-x}};$$

$$\text{Д)} \int_0^{\pi/2} \frac{dx}{2 - \sin x};$$

$$\text{e)} \int_{2\pi}^{3\pi} e^{2x} \cos^2 2x dx;$$

$$\text{Ж)} \int_{\pi/3}^{3\pi/2} (x-1) \left| \cos \frac{x}{2} \right| dx;$$

$$\text{И)} \int_0^{2\pi} (3x^2 + 5) \cos 2x dx;$$

$$\text{Л)} \int_0^{\pi/2} \frac{\cos x dx}{5+4 \cos x};$$

$$\text{Н)} \int_0^{2\pi} \sin^2 x \cos^6 x dx;$$

$$\text{П)} \int_0^1 \frac{x^4 dx}{(2-x^2)^{3/2}}.$$

$$\text{З)} \int_0^{3/2} \frac{[2x-1]}{\sqrt{1+x^2}} dx;$$

$$\text{К)} \int_0^1 \frac{x dx}{x^4+1};$$

$$\text{М)} \int_{-\arctg \frac{1}{3}}^0 \frac{3 \operatorname{tg} x + 1}{2 \sin 2x - 5 \cos 2x + 1} dx;$$

$$\text{О)} \int_1^8 \frac{5\sqrt{x+24} dx}{(x+24)^2 \sqrt{x}};$$

10.

$$\text{а)} \int_{2,5}^5 \frac{\sqrt{(25-x^2)^3}}{x^4} dx;$$

$$\text{В)} \int_{-2}^{-2/\sqrt{3}} \frac{dx}{x\sqrt{x^2-1}};$$

$$\text{Д)} \int_0^1 \frac{e^{2x} + 2e^x}{e^{2x} + 1} dx;$$

$$\text{Ж)} \int_{1/2}^3 \frac{|x^2 - 3x + 2|}{x^3} dx;$$

$$\text{И)} \int_0^{2\pi} (2x^2 - 15) \cos 3x dx;$$

$$\text{Л)} \int_0^{2\pi/3} \frac{1 + \sin x}{1 + \cos x + \sin x} dx;$$

$$\text{Н)} \int_0^{2\pi} \cos^8 \frac{x}{4} dx;$$

$$\text{П)} \int_0^2 \frac{x^2 dx}{\sqrt{16-x^2}}.$$

$$\text{б)} \int_0^3 \arcsin \sqrt{\frac{x}{1+x}} dx;$$

$$\text{г)} \int_0^{\pi/4} \cos^3 x dx;$$

$$\text{е)} \int_{2\pi}^{3\pi} e^{2x} \sin^2 2x dx;$$

$$\text{з)} \int_{-\pi/2}^{\pi/2} \operatorname{tg} \frac{x}{2} \operatorname{sgn} \left(\sin \frac{x}{3} \right) dx;$$

$$\text{к)} \int_{\sqrt{3}}^{\sqrt{8}} \frac{x + \frac{1}{x}}{\sqrt{x^2+1}} dx;$$

$$\text{м)} \int_{\pi/4}^{\arctg 3} \frac{1 + \operatorname{ctg} x}{(\sin x + 2 \cos x)^2} dx;$$

$$\text{о)} \int_1^2 \frac{x + \sqrt{3x-2} - 10}{\sqrt{3x-2} + 7} dx;$$

11.

a) $\int_0^{-\ln 2} \sqrt{1 - e^{2x}} dx;$

b) $\int_{-\sqrt{2}}^{-2/\sqrt{3}} \frac{dx}{x\sqrt{x^2-1}};$

в) $\int_1^9 x\sqrt[3]{1-x} dx;$

г) $\int_{-1}^2 |x^3|e^{-2x^3|x|+1} dx;$

д) $\int_0^{2\pi} (3 - 7x^2) \cos 2x dx;$

е) $\int_{\pi/3}^{\pi/2} \frac{\cos x dx}{1 + \sin x - \cos x};$

ж) $\int_0^{\pi} 2^4 \sin^8 \frac{x}{2} dx;$

з) $\int_0^2 \sqrt{4-x^2} dx.$

б) $\int_0^1 \arccos x dx;$

г) $\int_0^{1/3} \operatorname{ch}^2 3x dx;$

е) $\int_0^{3\pi} e^{-2x} \cos^2 2x dx;$

з) $\int_{-2}^0 \sqrt{\{x\}} dx;$

к) $\int_{\sqrt{3}}^{\sqrt{8}} \frac{x - \frac{1}{x}}{\sqrt{x^2+1}} dx;$

м) $\int_{\pi/4}^{\arccos \frac{1}{\sqrt{3}}} \frac{\operatorname{tg} x dx}{\sin^2 x - 5 \cos^2 x + 4};$

о) $\int_6^{10} \sqrt{\frac{4-x}{x-12}} dx;$

12.

a) $\int_0^1 \frac{dx}{(x+1)\sqrt{x^2+2}};$

b) $\int_1^2 \operatorname{arctg}(\sqrt[3]{x+1}) dx;$

в) $\int_1^e \frac{\sin(\ln x)}{x} dx;$

г) $\int_{-3}^3 \sqrt{|2-|x||} dx;$

д) $\int_0^{2\pi} (1 - 8x^2) \cos 4x dx;$

б) $\int_{-1/2}^2 \ln(2x^2 + 3) dx;$

г) $\int_{-1}^{1/2} \frac{dx}{\sqrt{2+x+3x^2}};$

е) $\int_0^{\pi} e^{x/3} \cos^2 x dx;$

з) $\int_{-\pi/4}^{\pi/3} \operatorname{sgn}(\operatorname{tg}^5 x) \sin^2 x dx;$

к) $\int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{x^2-1}};$

$$\text{Л)} \int_0^{\pi/2} \frac{1+\cos x}{1+\sin x+\cos x} dx;$$

$$\text{Н)} \int_0^4 2^8 \sin^6 x \cos^2 x dx;$$

$$\text{П)} \int_0^{-\pi/4} \frac{dx}{(16+x^2)^{3/2}}.$$

$$\text{М)} \int_0^{\pi/4} \frac{6 \sin^2 x dx}{3 \cos 2x-4};$$

$$\text{О)} \int_0^2 \frac{4\sqrt{2-x}-\sqrt{2x+2}}{(\sqrt{2x+2}+4\sqrt{2-x})(2x+2)^2} dx;$$

13.

$$\text{а)} \int_a^{2a} \frac{\sqrt{x^2-a^2}}{x^4} dx;$$

$$\text{В)} \int_{-4}^{-\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}};$$

$$\text{Д)} \int_1^e x^n \ln x dx \quad (n \in \mathbb{N});$$

$$\text{Ж)} \int_{-\pi/2}^{2\pi/3} \frac{|\operatorname{tg}^3 \frac{x}{2} - \operatorname{tg}^2 \frac{x}{2}|}{\cos^2 \frac{x}{2}} dx;$$

$$\text{И)} \int_{-1}^0 (x^2 + 2x + 1) \sin 3x dx;$$

$$\text{Л)} \int_0^{\pi/2} \frac{\sin x dx}{1+\sin x+\cos x};$$

$$\text{Н)} \int_{\pi/2}^{2\pi} 2^8 \sin^4 x \cos^4 x dx;$$

$$\text{П)} \int_0^4 x^2 \sqrt{16-x^2} dx.$$

$$\text{б)} \int_{1/e}^e |\ln x| dx;$$

$$\text{Г)} \int_0^{\pi/4} \frac{dx}{1+2 \sin^2 x};$$

$$\text{е)} \int_{4\pi}^{5\pi} e^{2x} \cos^2 2x dx;$$

$$\text{з)} \int_0^5 \{\sqrt{x}\} dx;$$

$$\text{к)} \int_0^{\sqrt{3}} \frac{x - \operatorname{arctg}^4 x}{1+x^2} dx;$$

$$\text{М)} \int_0^{\operatorname{arctg} 3} \frac{4+\operatorname{tg} x}{2 \sin^2 x + 18 \cos^2 x} dx;$$

$$\text{о)} \int_{-1/2}^0 \frac{x dx}{2+\sqrt{2x+1}};$$

14.

$$\text{а)} \int_0^{3/4} \frac{dx}{(x+1)\sqrt{x^2+1}};$$

$$\text{В)} \int_{-5}^{-2/\sqrt{3}} \frac{dx}{x\sqrt{x^2-1}};$$

$$\text{б)} \int_1^2 x^n \ln x dx \quad (n \in \mathbb{N});$$

$$\text{Г)} \int_{\pi/4}^{\pi/3} \frac{x dx}{\sin^2 x};$$

$$д) \int_{-1}^1 \frac{x dx}{x^2+x+1};$$

$$ж) \int_{-\pi/4}^{3\pi/4} |x \sin 2x| dx;$$

$$и) \int_0^3 (x^2 - 3x) \sin 2x dx;$$

$$л) \int_0^{2 \operatorname{arctg} \frac{1}{2}} \frac{1+\sin x}{(1-\sin x)^2} dx;$$

$$н) \int_0^{\pi} 2^4 \sin^2 x \cos^6 x dx;$$

$$п) \int_0^{5/2} \frac{x^2 dx}{\sqrt{25-x^2}}.$$

$$е) \int_{4\pi}^{5\pi} e^{2x} \sin^2 2x dx;$$

$$з) \int_{1/(2\pi)}^{2/\pi} \operatorname{sgn} \left(\sin \frac{1}{x} \right) \frac{dx}{x};$$

$$к) \int_0^1 \frac{x^3 dx}{x^2+1};$$

$$м) \int_0^{\operatorname{arctg} 2} \frac{12+\operatorname{tg} x}{3 \sin^2 x+12 \cos^2 x} dx;$$

$$о) \int_0^4 e^{\sqrt{\frac{4-x}{4+x}}} \frac{dx}{(4+x)\sqrt{16-x^2}};$$

15.

$$а) \int_0^{\ln 2} \sqrt[3]{e^x - 1} dx;$$

$$в) \int_{-4}^{-3} \frac{dx}{x\sqrt{x^2-1}};$$

$$д) \int_1^4 \arcsin \sqrt{\frac{x-1}{x}} dx;$$

$$ж) \int_{1-e}^{1/e} |\ln |x - 1|| dx;$$

$$и) \int_0^{\pi} (x^2 - 3x + 2) \sin x dx;$$

$$л) \int_0^{\pi/2} \frac{\cos x dx}{1+\sin x+\cos x};$$

$$н) \int_0^{2\pi} \cos^8 x dx;$$

$$п) \int_0^5 x^2 \sqrt{25 - x^2} dx.$$

$$б) \int_{\frac{1}{\pi/2}}^3 \operatorname{arctg} \sqrt{x+2} dx;$$

$$г) \int_0^{\pi/2} \frac{dx}{1+\sin x+\cos x};$$

$$е) \int_{4\pi}^{5\pi} e^{-2x} \cos^2 2x dx;$$

$$з) \int_{1/2}^{\sqrt{3}} \{x^2\} \frac{dx}{x};$$

$$к) \int_0^{\sin 1} \frac{\arcsin^2 x+1}{\sqrt{1-x^2}} dx;$$

$$м) \int_0^{\operatorname{arctg} \frac{2}{3}} \frac{6+\operatorname{tg} x}{9 \sin^2 x+4 \cos^2 x} dx;$$

$$о) \int_{1/8}^1 \frac{15\sqrt{x+3}}{(x+3)^2\sqrt{x}} dx;$$

16.

а) $\int_0^1 x^2 \sqrt{1-x^2} dx;$

б) $\int_{-5}^{-4} \frac{dx}{x\sqrt{x^2-1}};$

в) $\int_0^{\sqrt{3}} x \operatorname{arctg} x dx;$

г) $\int_{-e}^{-1/e} \frac{\sqrt{|\ln|x||}}{x} dx;$

д) $\int_0^{\pi/2} (x^2 - 5x + 6) \sin 3x dx;$

е) $\int_0^{2 \operatorname{arctg} \frac{1}{3}} \frac{\cos x dx}{(1-\sin x)(1+\cos x)};$

ж) $\int_0^{2\pi} \sin^8 \frac{x}{4} dx;$

з) $\int_0^4 \sqrt{16-x^2} dx.$

б) $\int_{-1}^0 \arcsin \sqrt{x+1} dx;$

в) $\int_{\pi/6}^{\pi/3} \operatorname{tg}^4 x dx;$

г) $\int_{4\pi}^{5\pi} e^{-2x} \sin^2 2x dx;$

д) $\int_{1/4}^2 x \left[\frac{1}{x}\right]^2 dx;$

е) $\int_1^3 \frac{1-\sqrt{x}}{\sqrt{x}(x+1)} dx;$

ж) $\int_0^{\arcsin \sqrt{3/7}} \frac{\operatorname{tg}^2 x dx}{3 \sin^2 x + 4 \cos^2 x - 7};$

з) $\int_{-5/3}^1 \frac{\sqrt[3]{3x+5}+2}{1+\sqrt[3]{3x+5}} dx;$

17.

а) $\int_0^{1/\sqrt{3}} \frac{dx}{(5x^2+1)\sqrt{x^2+1}};$

б) $\int_{3/2}^2 \frac{dx}{x\sqrt{x^2-1}};$

в) $\int_0^{\pi/2} \frac{dx}{4+\sin x};$

г) $\int_{-1}^1 \sqrt{|(1-2x)^5|} dx;$

б) $\int_{-1}^0 \arcsin(x+1) dx;$

в) $\int_0^1 \frac{x^3+2x^2+3x+4}{x^3+x^2+x+1} dx;$

г) $\int_{2\pi}^{3\pi} e^{-2x} \sin^2 2x dx;$

д) $\int_{-8}^0 \{\sqrt{|x|}\} dx;$

$$\text{И)} \int_{-3}^0 (x^2 + 6x + 9) \sin 2x \, dx;$$

$$\text{К)} \int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{x\sqrt{x^2+1}};$$

$$\text{Л)} \int_{-2\pi/3}^0 \frac{\cos x \, dx}{1+\cos x - \sin x};$$

$$\text{М)} \int_0^{\pi/4} \frac{7+3 \operatorname{tg} x}{(\sin x + 2 \cos x)^2} \, dx;$$

$$\text{Н)} \int_0^{\pi} 2^4 \sin^6 \frac{x}{2} \cos^2 \frac{x}{2} \, dx;$$

$$\text{О)} \int_2^3 \sqrt{\frac{3-2x}{2x-7}} \, dx;$$

$$\text{П)} \int_0^{4\sqrt{3}} \frac{dx}{\sqrt{(64-x^2)^3}}.$$

18.

$$\text{а)} \int_0^3 \sqrt{3x - x^2} \, dx;$$

$$\text{б)} \int_{-\pi}^{\pi} e^{x+1} \sin x \, dx;$$

$$\text{в)} \int_{1/3}^{1/2} \frac{dx}{x\sqrt{1-x^2}};$$

$$\text{г)} \int_{-3}^{-2} \frac{dx}{\sqrt{x^2+6x+10}};$$

$$\text{д)} \int_1^{e^{\pi/4}} \frac{\operatorname{tg}(\ln x)}{x} \, dx;$$

$$\text{е)} \int_{2\pi}^{3\pi} e^{x/3} \sin^2 x \, dx;$$

$$\text{ж)} \int_{-e/4}^{-1/4} |\ln |4x|| \, dx;$$

$$\text{з)} \int_{1/(2\pi)}^{4/\pi} \operatorname{sgn} \left(\cos \frac{1}{x} \right) \frac{dx}{x};$$

$$\text{и)} \int_0^{\pi/4} (x^2 + 17,5) \sin 2x \, dx;$$

$$\text{к)} \int_1^e \frac{1+\ln x}{x} \, dx;$$

$$\text{л)} \int_{-\pi/2}^0 \frac{\cos x \, dx}{(1+\cos x - \sin x)^2};$$

$$\text{м)} \int_{\arcsin \frac{2}{\sqrt{5}}}^{\arcsin \frac{3}{\sqrt{10}}} \frac{2 \operatorname{tg} x + 5}{(5 - \operatorname{tg} x) \sin 2x} \, dx;$$

$$\text{н)} \int_{-\pi/2}^0 2^8 \sin^4 x \cos^4 x \, dx;$$

$$\text{о)} \int_0^7 \frac{\sqrt{x+25}}{(x+25)^2 \sqrt{x+1}} \, dx;$$

$$\text{п)} \int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2-2}}{x^4} \, dx.$$

19.

$$a) \int_{-\ln\sqrt{3}}^{\ln\sqrt{3}} \frac{e^{2x}+3e^x}{e^{2x}+1} dx;$$

$$b) \int_{\sqrt{3}}^2 \frac{dx}{x\sqrt{x^2-1}};$$

$$c) \int_{-\pi/2}^{\pi/2} \frac{dx}{2+\sin x+\cos x};$$

$$d) \int_{-\pi/3}^{\pi/2} (x-1) \left| \sin \frac{x}{2} \right| dx;$$

$$e) \int_0^{\pi/2} (1-5x^2) \sin x dx;$$

$$f) \int_0^{\pi/2} \frac{\cos x dx}{(1+\cos x+\sin x)^2};$$

$$g) \int_{\pi/2}^{\pi} 2^8 \sin^2 x \cos^6 x dx;$$

$$h) \int_0^{2\sqrt{2}} \frac{x^4 dx}{(16-x^2)\sqrt{16-x^2}}.$$

$$a) \int_0^{1/2} x \arcsin x dx;$$

$$b) \int_0^{1/2} \frac{dx}{\sqrt{3+4x-4x^2}};$$

$$c) \int_0^{\pi} e^{-2x} \cos^2 2x dx;$$

$$d) \int_{1/2}^{\sqrt{2}} \left\{ x^3 \right\} \frac{dx}{x};$$

$$e) \int_0^{\sqrt{3}} \frac{\operatorname{arctg} x+x}{1+x^2} dx;$$

$$f) \int_{-\arccos \frac{1}{\sqrt{10}}}^0 \frac{3 \operatorname{tg}^2 x-50}{2 \operatorname{tg} x+7} dx;$$

$$g) \int_0^2 \frac{4\sqrt{2-x}-\sqrt{3x+2}}{(\sqrt{3x+2}+4\sqrt{2-x})(3x+2)^2} dx;$$

20.

$$a) \int_0^5 x^2 \sqrt{25-x^2} dx;$$

$$b) \int_{1/4}^{1/3} \frac{dx}{x\sqrt{1-x^2}};$$

$$c) \int_1^2 \frac{dx}{x \cos^2(\ln x)};$$

$$d) \int_{1/2}^4 \frac{|x^2-4x+3|}{x^3} dx;$$

$$a) \int_{-1}^3 \arcsin \sqrt{\frac{x+1}{x+2}} dx;$$

$$b) \int_1^3 \frac{dx}{\sqrt{x^2+2x-1}};$$

$$c) \int_0^{\pi} e^{-2x} \cos^2 2x dx;$$

$$d) \int_{1/\sqrt{5}}^2 x^2 \left[\frac{1}{x} \right]^2 dx;$$

$$\text{И)} \int_{\pi/4}^3 (3x - x^2) \sin 2x dx;$$

$$\text{Л)} \int_0^{2 \operatorname{arctg} \frac{1}{2}} \frac{1 - \sin x}{\cos x(1 + \cos x)} dx;$$

$$\text{Н)} \int_0^{\pi} 2^4 \cos^8 x dx;$$

$$\text{П)} \int_{-3}^3 x^2 \sqrt{9 - x^2} dx.$$

$$\text{К)} \int_1^e \frac{x^2 + \ln x^2}{x} dx;$$

$$\text{М)} \int_0^{\pi/4} \frac{5 \operatorname{tg} x + 2}{2 \sin 2x + 5} dx;$$

$$\text{О)} \int_0^2 e^{\sqrt{\frac{2-x}{2+x}}} \frac{dx}{(2+x)\sqrt{4-x^2}};$$

21.

$$\text{а)} \int_2^4 \frac{\sqrt{x^2 - 4}}{x^4} dx;$$

$$\text{В)} \int_{\sqrt{3/2}}^{\sqrt{3}} \frac{dx}{x\sqrt{x^2 - 1}};$$

$$\text{Д)} \int_{-\pi/4}^{\pi/4} x^2 (\operatorname{arctg}^3 x + \cos 2x) dx;$$

$$\text{Ж)} \int_{-1}^2 |x^3| 2^{x^3|x|^{-2}} dx;$$

$$\text{И)} \int_1^2 x \ln^2 x dx;$$

$$\text{Л)} \int_0^{\pi/2} \frac{\sin x dx}{(1 + \sin x)^2};$$

$$\text{Н)} \int_0^{2\pi} \sin^8 x dx;$$

$$\text{П)} \int_1^{\sqrt{3}} \frac{dx}{\sqrt{(1+x^2)^3}}.$$

$$\text{б)} \int_{-1}^1 \ln(x^2 + 1) dx;$$

$$\text{Г)} \int_0^{\pi/2} \frac{dx}{1 + 2 \sin x + \cos x};$$

$$\text{е)} \int_0^{\pi} e^{x/3} \sin^2 2x dx;$$

$$\text{з)} \int_{-3/2}^1 (2 + |x|)\{x\} dx;$$

$$\text{К)} \int_0^1 \frac{x dx}{\sqrt{x^4 + x^2 + 1}};$$

$$\text{М)} \int_{\pi/4}^{\operatorname{arcsin} \frac{2}{\sqrt{5}}} \frac{4 \operatorname{tg} x - 5}{4 \cos^2 x - \sin 2x + 1} dx;$$

$$\text{О)} \int_3^5 \sqrt{\frac{2-x}{x-6}} dx;$$

22.

$$a) \int_0^{\sqrt{3}} x^3 \sqrt{x^2 + 3} dx;$$

$$б) \int_{-1/\sqrt{3}}^{\sqrt{3}} x^3 \operatorname{arctg} x dx;$$

$$B) \int_{1/9}^{1/5} \frac{dx}{x\sqrt{1-x^2}};$$

$$Г) \int_0^1 \frac{x^3+3x^2+6x+9}{x^3+x^2+x+1} dx;$$

$$Д) \int_0^{\ln 2} \frac{e^{2x}+1}{e^{2x}+e^x} dx;$$

$$e) \int_0^{\pi} e^{x/3} \cos^2 2x dx;$$

$$Ж) \int_{-2}^2 \sqrt{|1-2|x||} dx;$$

$$з) \int_{\pi/4}^{5\pi/4} \sin^2 x \operatorname{sgn}(\cos x) dx;$$

$$И) \int_1^{e^2} \frac{\ln^2 x}{\sqrt{x}} dx;$$

$$К) \int_0^1 \frac{x^3 dx}{(x^2+1)^2};$$

$$Л) \int_0^{\pi/2} \frac{\sin x}{(1+\cos x+\sin x)^2} dx;$$

$$М) \int_0^{\arcsin \sqrt{7/8}} \frac{6 \sin^2 x}{4+3 \cos 2x} dx;$$

$$Н) \int_0^{2\pi} \sin^6 \frac{x}{4} \cos^2 \frac{x}{4} dx;$$

$$o) \int_{1/24}^{1/3} \frac{5\sqrt{x+1}}{(x+1)^2 \sqrt{x}} dx;$$

$$П) \int_0^2 \frac{dx}{\sqrt{(16-x^2)^3}}.$$

4. Найти непрерывную обобщенную первообразную функции:

$$1. y = \operatorname{sgn}((x-2)(x^2+2x)). \quad 2. y = \left[\frac{2x}{|x|+1} \right].$$

$$3. y = \operatorname{sgn}((x+2)(x^2-2x)). \quad 4. y = \left[\frac{2x}{2+|x|} \right].$$

$$5. y = \left[\frac{x}{1+\frac{|x|}{2}} \right]. \quad 6. y = \operatorname{sgn}(x(2x^2-8)).$$

$$7. y = \operatorname{sgn}(x^3(1-x^2)). \quad 8. y = \left[\frac{\sqrt{2}x}{\sqrt{1+x^2}} \right].$$

9. $y = \left[\frac{2x}{\sqrt{3+x^2}} \right]$.

10. $y = \operatorname{sgn}((3+x)(3x-x^2))$.

11. $y = \operatorname{sgn}((3-x)(x^2+3x))$.

12. $y = \left[\frac{4x}{\sqrt{1+4x^2}} \right]$

13. $y = \left[\frac{3}{2+|x|} \right]$.

14. $y = \operatorname{sgn}(x^5(x^2-1))$.

15. $y = \operatorname{sgn}(x(12-3x^2))$.

16. $y = \operatorname{sgn}(x^7(1-x^2))$.

17. $y = \left[\frac{4x}{2|x|+1} \right]$.

18. $y = \operatorname{sgn}((x-3)(x^2+3x))$.

19. $y = \operatorname{sgn}(x^3(4x^2-1))$.

20. $y = \left[\frac{\sqrt{3}x}{\sqrt{1+3x^2}} \right]$.

21. $y = \left[\frac{4}{3+|x|} \right]$.

22. $y = \operatorname{sgn}(x^9(x^2-9))$.