

БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
МЕХАНИКО-МАТЕМАТИЧЕСКИЙ ФАКУЛЬТЕТ
Кафедра теории функций

МАТЕМАТИЧЕСКИЙ АНАЛИЗ

ИНДИВИДУАЛЬНЫЕ ЗАДАНИЯ
для студентов механико-математического факультета

МИНСК
БГУ
2012

ТРЕБОВАНИЯ К ОФОРМЛЕНИЮ ИНДИВИДУАЛЬНЫХ ЗАДАНИЙ

Студент выполняет индивидуальные задания в отдельной тетради, оставляя поля для замечаний преподавателя.

На обложке тетради студент указывает свою фамилию, имя, номер учебной группы и вариант индивидуального задания.

Решения задач следует излагать в порядке номеров, указанных в задании.

Решения задач излагать **подробно и аккуратно, выполняя все необходимые теоретические обоснования.**

ИНДИВИДУАЛЬНОЕ ЗАДАНИЕ 17 «НЕСОБСТВЕННЫЕ ИНТЕГРАЛЫ. СОБСТВЕННЫЕ ИНТЕГРАЛЫ, ЗАВИСЯЩИЕ ОТ ПАРАМЕТРА»

1. Вычислить интеграл:

$$1. \int_0^1 \frac{x^4}{\sqrt[6]{(1-x^5)^5}} dx.$$

$$3. \int_2^3 \frac{x}{\sqrt[5]{(x^2-4)^3}} dx.$$

$$5. \int_1^2 \frac{1}{x\sqrt{\ln x}} dx.$$

$$7. \int_0^2 \frac{x^3}{\sqrt{4-x^2}} dx.$$

$$9. \int_0^{+\infty} \frac{dx}{1+x^3}.$$

$$11. \int_1^{+\infty} \frac{\ln(x^2+1)}{x^2} dx.$$

$$13. \int_0^{+\infty} x^2 e^{-x/2} dx.$$

$$15. \int_1^{+\infty} \frac{dx}{x\sqrt{x^2+1}}.$$

$$17. \int_0^1 \frac{dx}{(2-x)\sqrt{1-x}}.$$

$$19. \int_0^{1/e} \frac{1}{x \ln^3 x} dx.$$

$$21. \int_1^e \frac{1}{x \sqrt[3]{\ln x}} dx.$$

$$2. \int_4^{+\infty} \frac{dx}{x \ln^3 x}.$$

$$4. \int_{-\infty}^{+\infty} \frac{dx}{x^2+2x+2}.$$

$$6. \int_1^{+\infty} \frac{x^2}{1+x^6} dx.$$

$$8. \int_1^{+\infty} \frac{dx}{(1+x)\sqrt{x}}.$$

$$10. \int_0^1 \frac{x^2}{\sqrt[4]{(1-x^3)^3}} dx.$$

$$12. \int_2^3 \frac{x^3}{\sqrt[5]{(x^4-16)^3}} dx.$$

$$14. \int_0^{1/2} \frac{1}{x \ln^2 x} dx.$$

$$16. \int_1^5 \frac{x}{\sqrt[3]{(x^2-1)^2}} dx.$$

$$18. \int_2^{+\infty} \frac{dx}{1-x^3}.$$

$$20. \int_2^3 \frac{x}{\sqrt[7]{(9-x^2)^5}} dx.$$

$$22. \int_1^{\sqrt{2}} \frac{x^5}{\sqrt[5]{(8-x^6)^3}} dx.$$

2. Вычислить интеграл:

1. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2-12x+35}$.

3. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2+3x-28}$.

5. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2-10x+21}$.

7. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2+5x-14}$.

9. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2-11x+30}$.

11. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2+2x-24}$.

13. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2-9x+18}$.

15. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2+4x-12}$.

17. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2-13x+40}$.

19. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2+4x-32}$.

21. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{x^2-11x+24}$.

2. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{-45-4x+x^2}$.

4. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{36+13x+x^2}$.

6. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{-27-6x+x^2}$.

8. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{18+11x+x^2}$.

10. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{-50-5x+x^2}$.

12. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{40+14x+x^2x^2x}$.

14. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{-30-7x+x^2}$.

16. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{20+12x+x^2x^2x}$.

18. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{-55-6x+x^2}$.

20. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{44+15x+x^2}$.

22. v.p. $\int_{-\infty}^{+\infty} \frac{dx}{-33-8x+x^2}$.

3. Исследовать интеграл на сходимость:

1. $\int_2^{+\infty} \frac{dx}{x^k \ln x}$.
2. $\int_0^{\pi/2} \sin^p x \cdot \cos^q x dx$.
3. $\int_0^{+\infty} x^\alpha \operatorname{arctg}(\beta x) dx$. ($\beta \neq 0$)
4. $\int_0^1 \frac{x \sin^p x}{\sqrt{1-x^2}} dx$.
5. $\int_0^{+\infty} x^\alpha e^{\beta x} dx$.
6. $\int_0^{\pi/2} \frac{\sin^\alpha x \cdot \cos^\beta x}{\sqrt{1-\cos x}} dx$.
7. $\int_0^{+\infty} \frac{dx}{\sqrt{1+x^2}(x^m+x^n)}$.
8. $\int_0^1 \frac{\ln x}{1-x^2} dx$.
9. $\int_0^{\pi/4} \frac{\ln(1+x)}{\sqrt[3]{\left(\frac{\pi^2}{16}-x^2\right)} \sqrt{\sin x}} dx$.
10. $\int_1^{+\infty} x^\alpha \ln^\beta x dx$.
11. $\int_0^1 \frac{x^p}{\sqrt{1-x^4}} dx$.
12. $\int_0^{+\infty} \frac{x^q}{1+x^p} \operatorname{arctg}^2 x dx$. ($p \geq 0$)
13. $\int_0^{\pi/2} \frac{\sqrt{\sin x}(1-\cos x)}{x^m \sqrt[3]{\cos^2 x}} dx$.
14. $\int_0^{+\infty} \frac{x^\mu \sqrt{\operatorname{arctg}(x^3)}}{\sqrt[3]{1+x^4}} dx$.
15. $\int_0^1 \frac{dx}{\sqrt[3]{x(e^x-e^{-x})}}$.
16. $\int_1^{+\infty} \frac{\ln x}{x\sqrt{x^2-1}} dx$.
17. $\int_0^{\pi/2} \frac{(1-\cos x) \cdot \cos^\alpha x}{\operatorname{arctg}^\beta x} dx$.
18. $\int_0^1 \frac{x^\alpha}{\sqrt[3]{1-x^3}} dx$.
19. $\int_0^1 \frac{\operatorname{arctg}^\beta x}{x^2 \sqrt{1-x^2}} dx$.
20. $\int_0^{+\infty} \frac{x^\beta}{\operatorname{arctg}(\alpha x)} dx$. ($\alpha \neq 0$)
21. $\int_1^{+\infty} \frac{x^\alpha}{\ln^\beta x} dx$.
22. $\int_0^{+\infty} \frac{e^{\alpha x}}{x^\beta} dx$.

4. Исследовать интеграл на абсолютную и условную сходимось:

1.
$$\int_0^{+\infty} \frac{\sin(x^2)}{x} dx.$$

2.
$$\int_{\pi/6}^{+\infty} \frac{\cos x}{\sqrt{x^2+1}} dx.$$

3.
$$\int_0^{+\infty} \sin(x^2) dx.$$

4.
$$\int_0^{+\infty} \frac{\sqrt{x} \cos x}{x+\pi} dx.$$

5.
$$\int_0^{+\infty} \frac{\sin 3x}{x} dx.$$

6.
$$\int_1^{+\infty} \frac{\ln^2 x}{x} \cos x dx.$$

7.
$$\int_0^{+\infty} \frac{\sqrt{x} \sin x}{1+x} dx.$$

8.
$$\int_0^{+\infty} \cos(x^2) dx.$$

9.
$$\int_0^{+\infty} \frac{\sin x}{\sqrt[3]{x+2}} dx.$$

10.
$$\int_0^{+\infty} x \cos(e^x) dx.$$

11.
$$\int_0^{+\infty} \frac{\sqrt[5]{x} \sin x}{\sqrt[4]{x^3+3}} dx.$$

12.
$$\int_1^{+\infty} \frac{\cos(x^2)}{x} dx.$$

13.
$$\int_0^{+\infty} \frac{\sin(x^3)}{x^{3/4}} dx.$$

14.
$$\int_{\pi/2}^{+\infty} \frac{\cos 2x}{\sqrt[3]{x}} dx.$$

15.
$$\int_0^{+\infty} \sin(e^x) x^2 dx.$$

16.
$$\int_{-\pi/4}^{+\infty} \frac{\cos 3x}{\sqrt{x+1}} dx.$$

17.
$$\int_1^{+\infty} \frac{\ln \sqrt{x}}{x} \sin x dx.$$

18.
$$\int_1^{+\infty} \frac{\sin(x^3)}{x} dx.$$

19.
$$\int_0^{+\infty} x^3 \cos(e^x) dx.$$

20.
$$\int_{\pi/2}^{+\infty} \frac{\sin 3x}{\sqrt[5]{x+17}} dx.$$

21.
$$\int_1^{+\infty} \frac{\cos(x^4)}{\sqrt{x}} dx.$$

22.
$$\int_{\pi/2}^{+\infty} \frac{\sin x}{\sqrt[3]{x^3+x^2+x+1}} dx.$$

5. Найти предел:

$$1. \lim_{\beta \rightarrow 1} \int_0^{\pi/4} \frac{e^{(\beta^4-1)x^2+1+\beta^2}}{\cos^2\left(\beta x + \frac{\pi}{8}\right)} dx.$$

$$2. \lim_{\alpha \rightarrow 0} \int_0^1 \frac{(3x + \alpha \sqrt{x^3})}{\sqrt{1 + \sin(\alpha^2 x) + x^4}} dx.$$

$$3. \lim_{q \rightarrow 2} \int_{-2}^0 \frac{\sqrt[3]{\ln\left(x + \sqrt{\cos^2((4-q^2)x) + x^2}\right)}}{\sqrt{1+x^2}} dx.$$

$$4. \lim_{\rho \rightarrow 0} \int_{-1}^1 \frac{(\sin(\rho x) + x^2)}{e^{\rho x^2 + 2|x|^3} + e^{-2|x|^3}} dx.$$

$$5. \lim_{\beta \rightarrow 1} \int_{1/2}^1 \frac{\left(\cos \frac{\beta\pi}{2} + x\right) \ln(2+x^2 - \sqrt{\beta})}{\beta^2 + x^2} dx.$$

$$6. \lim_{c \rightarrow \sqrt{2}} \int_{-2}^{-1} e^{\sqrt{2-c^2+x^2}} \left(\cos \frac{c\pi}{2\sqrt{2}} + c^4 x\right) dx.$$

$$7. \lim_{\alpha \rightarrow 1} \int_0^{\pi/6} \frac{\operatorname{arctg}\left(\sqrt[3]{\alpha^2-1} \cdot x + 1\right)}{\sin^2\left(\frac{\pi}{4} - x\right)} dx.$$

$$8. \lim_{p \rightarrow 0} \int_0^{1/2} \frac{\operatorname{tg}\left(\frac{p \sqrt[3]{x} - \pi}{4}\right)}{\sqrt[4]{2-x + \sin(px^2)}} dx.$$

$$9. \lim_{\beta \rightarrow 1} \int_{-1/2}^{1/2} \frac{\left(\beta x^2 + \cos \frac{\beta\pi}{2}\right) \arcsin\left((2-\beta)|x|^3\right)}{\sqrt{\operatorname{ch}\left((\beta^2-1)x\right) - x^6}} dx.$$

$$10. \lim_{\alpha \rightarrow 0} \int_1^2 \frac{\sin\left(\frac{\pi}{2} + \alpha x\right)}{\alpha^3 + \ln(1 + \alpha^2 x) + x^3 + 1} dx.$$

$$11. \lim_{p \rightarrow \pi} \int_{-1/4}^{1/4} \frac{e^{(\pi^2-p^2)x+p}}{\sqrt{\operatorname{tg} \frac{p}{4} + \sin((p-\pi)x) + e^{2|x|}}} dx.$$

$$12. \lim_{\beta \rightarrow 2} \int_{-1}^{-1/2} \frac{\ln(3+x^4-\sqrt{2\beta}) \cdot \left(\beta x^3 + \operatorname{sh} \frac{(\beta-2)x}{2}\right)}{\beta^2 - 3 + x^4} dx.$$

$$13. \lim_{\alpha \rightarrow 0} \int_{-1}^1 \frac{\sqrt{x^4 + \alpha^2 |x|^3} \cos(\alpha x)}{\sqrt{2 + x^6 + \operatorname{tg}^2(\alpha \sqrt{|x|})}} dx.$$

$$14. \lim_{\beta \rightarrow \sqrt{2}} \int_{-1/2}^{1/2} \frac{\sin\left((\beta^2-1)\frac{\pi}{6}\right)}{\sqrt[3]{\arccos((3-\beta^2)|x|)} \cdot \sqrt{e^{\beta^2-2-x^2} + \cos \frac{\pi\beta}{2\sqrt{2}}}} dx.$$

$$15. \lim_{\alpha \rightarrow 1} \int_0^{\pi/6} \frac{\sqrt{\alpha} + \operatorname{tg}^3\left(\arcsin \frac{\alpha}{2} + \alpha x\right)}{\cos^2\left(\sqrt[3]{\alpha} \cdot x + \frac{\pi}{6}\right)} dx.$$

$$16. \lim_{\rho \rightarrow 0} \int_1^2 \frac{\left(e^{-\sin \rho^2} + \operatorname{arctg}(\rho \sqrt{x} + x^2)\right) x}{x^4 \operatorname{tg} \frac{(\rho+1)\pi}{4} + \operatorname{ch} \frac{\rho}{x}} dx.$$

$$17. \lim_{c \rightarrow \sqrt{3}} \int_{-3}^{-2} \left(c^2 x - \sin \frac{c\pi\sqrt{3}}{3}\right) e^{\sqrt{c^2+x^2-3}} dx.$$

$$18. \lim_{\beta \rightarrow 2} \int_1^3 \frac{\ln(6+x^2-\beta) \cdot \left(2x + \sin \frac{\beta\pi}{2}\right)}{x^2 + \beta^2 + \ln(\beta-1)} dx.$$

$$19. \lim_{\rho \rightarrow 0} \int_{-2}^2 \frac{(3x^2 - \operatorname{sh}(\rho x^2))}{e^{\rho^2 x - 2|x|^3} + e^{2|x|^3}} dx.$$

$$20. \lim_{q \rightarrow 3} \int_{-3}^{-2} \frac{\sqrt[5]{\ln\left(\sqrt{x^2 - \operatorname{ch}^4\left((q^3-27)\sqrt{|x|}\right) + x}\right)}}{\sqrt{x^2-1}} dx.$$

$$21. \lim_{\alpha \rightarrow 0} \int_0^1 \frac{\left(7x + \alpha \sqrt{x^5 + \operatorname{arctg} x}\right)}{\sqrt{x^4 - \operatorname{sh}(\alpha x^3) + 1}} dx.$$

$$22. \lim_{\beta \rightarrow 2} \int_{\pi/4}^{\pi/2} \frac{2^{(\beta^3-8)x^3 + \beta^2 - 3}}{\sin^2\left(\frac{\pi}{4} + \beta^2 x\right)} dx.$$

6. Найти $F'(x)$, если $f \in C^1(\mathbb{R} \times \mathbb{R})$:

$$1. F(x) = \int_{\sin x}^x f(\sin x - t, x^2 - t) dt.$$

$$2. F(x) = \int_x^{e^x} f(e^x - t, x - t) dt.$$

$$3. F(x) = \int_x^{\sin x} f(\sin x - t, x - t) dt.$$

$$4. F(x) = \int_{x^2}^{e^x} f(e^x - t, t - x^2) dt.$$

$$5. F(x) = \int_{3^x}^{\sin x} f(\sin x - t, 3^x - t) dt.$$

$$6. F(x) = \int_{x^3}^{\operatorname{tg} x} f(\operatorname{tg} x - t, x^3 - t) dt.$$

$$7. F(x) = \int_{\operatorname{ctg} x}^{2x} f(t - 2x, \operatorname{ctg} x - t) dt.$$

$$8. F(x) = \int_{2^x}^{x^2} f(2^x - t, x^2 - t) dt.$$

$$9. F(x) = \int_{\cos x}^x f(xt - 1, \cos x - t) dt.$$

$$10. F(x) = \int_{\operatorname{tg} x}^x f(x + t, t \operatorname{ctg} x - 1) dt.$$

$$11. F(x) = \int_{e^x}^x f(x - t, t - e^x) dt.$$

$$12. F(x) = \int_{\cos x}^{\sin x} f(t - \sin x, \cos x - t) dt.$$

$$13. F(x) = \int_{\operatorname{sh} x}^{\cos x} f(t - \cos x, \operatorname{sh} x - t) dt.$$

$$14. F(x) = \int_{\sin x}^x f(x - t, t - \sin t) dt.$$

$$15. F(x) = \int_{\operatorname{ctg} x}^x f(t \cdot \operatorname{tg} x - 1, x - t) dt.$$

$$16. F(x) = \int_{\operatorname{tg} x}^{x^2} f(x^2 - t, t - \operatorname{tg} x) dt.$$

$$17. F(x) = \int_{e^x}^{\sin x} f(\sin x - t, e^x - t) dt.$$

$$18. F(x) = \int_{\sin x}^x f(\sin x - t, e^x - t) dt.$$

$$19. F(x) = \int_{\sin x}^{x^2} f(t - \sin x, x^2 - t) dt.$$

$$20. F(x) = \int_{\operatorname{tg} x}^{3^x} f(2 \cdot 3^x + t, t - \operatorname{tg} x) dt.$$

$$21. F(x) = \int_x^{2^x} f(tx - 1, 2^x - t) dt.$$

$$22. F(x) = \int_{\sin x}^{e^x} f(t - \sin x, t - e^x) dt.$$

7. Найти $F^{(4)}(x)$, если:

1. $F(x) = \frac{1}{6} \int_0^x (x-t)^3 e^{-2t} dt.$

2. $F(x) = \frac{1}{6} \int_0^x (x-t)^3 \cos t^2 dt.$

3. $F(x) = \frac{1}{6} \int_0^x (x-t)^3 e^{-t^2} dt.$

4. $F(x) = \frac{1}{6} \int_0^x (x-t)^3 \cos^3 t dt.$

5. $F(x) = \frac{1}{6} \int_0^x (t-x)^3 5^t dt.$

6. $F(x) = \frac{1}{6} \int_0^x (t-x)^3 \ln \left(t + \sqrt{1+t^2} \right) dt.$

7. $F(x) = \frac{1}{6} \int_0^x (t-x)^3 2^{-t^2} dt.$

8. $F(x) = \frac{1}{6} \int_0^x (x-t)^3 \operatorname{arctg} t dt.$

9. $F(x) = \frac{1}{6} \int_0^x (x-t)^3 e^{t^2} dt.$

10. $F(x) = \frac{1}{6} \int_0^x (t-x)^3 \sin t^2 dt.$

11. $F(x) = \frac{1}{6} \int_0^x (x-t)^3 \sin^3 t dt.$

$$12. F(x) = \frac{1}{6} \int_0^x (x-t)^3 2^t dt.$$

$$13. F(x) = \frac{1}{6} \int_0^x (t-x)^3 e^{-2t^2} dt.$$

$$14. F(x) = \frac{1}{6} \int_0^x (x-t)^3 \operatorname{ch} 3t dt.$$

$$15. F(x) = \frac{1}{6} \int_0^x (t-x)^3 3^{t^2} dt.$$

$$16. F(x) = \frac{1}{6} \int_0^x (x-t)^3 \operatorname{sh} 2t dt.$$

$$17. F(x) = \frac{1}{6} \int_0^x (x-t)^3 e^{t^3} dt.$$

$$18. F(x) = \frac{1}{6} \int_0^x (x-t)^3 \cos t^3 dt.$$

$$19. F(x) = \frac{1}{6} \int_0^x (t-x)^3 \cos^4 t dt.$$

$$20. F(x) = \frac{1}{6} \int_0^x (x-t)^3 7^t dt.$$

$$21. F(x) = \frac{1}{6} \int_0^x (t-x)^3 \operatorname{arctg}(t^2) dt.$$

$$22. F(x) = \frac{1}{6} \int_0^x (x-t)^3 \operatorname{ch} 5t dt.$$

8. Вычислить:

1.
$$\int_0^{\pi/2} \ln (a^2 \sin^2 x + 5 \cos^2 x) dx.$$

2.
$$\int_0^{\pi/2} \ln (7 \sin^2 x + b^2 \cos^2 x) dx.$$

3.
$$\int_0^{\pi/2} \ln (a^2 \sin^2 x + \cos^2 x) dx.$$

4.
$$\int_0^{\pi/2} \ln (2 \sin^2 x + b^2 \cos^2 x) dx.$$

5.
$$\int_0^{\pi/2} \ln (a^2 \sin^2 x + 9 \cos^2 x) dx.$$

6.
$$\int_0^{\pi/2} \ln (5 \sin^2 x + b^2 \cos^2 x) dx.$$

7.
$$\int_0^{\pi/2} \ln (a^2 \sin^2 x + 3 \cos^2 x) dx.$$

8.
$$\int_0^{\pi/2} \ln (2 \sin^2 x + a^2 \cos^2 x) dx.$$

9.
$$\int_0^{\pi/2} \ln (a^2 \sin^2 x + 7 \cos^2 x) dx.$$

10.
$$\int_0^{\pi/2} \ln (3 \sin^2 x + b^2 \cos^2 x) dx.$$

11.
$$\int_0^{\pi/2} \ln (a^2 \sin^2 x + 4 \cos^2 x) dx.$$

$$12. \int_0^{\pi/2} \ln (\cos^2 x + \beta^2 \sin^2 x) dx.$$

$$13. \int_0^{\pi/2} \ln (b^2 \sin^2 x + 8 \cos^2 x) dx.$$

$$14. \int_0^{\pi/2} \ln (9 \sin^2 x + \beta^2 \cos^2 x) dx.$$

$$15. \int_0^{\pi/2} \ln (a^2 \sin^2 x + 2 \cos^2 x) dx.$$

$$16. \int_0^{\pi/2} \ln (6 \sin^2 x + b^2 \cos^2 x) dx.$$

$$17. \int_0^{\pi/2} \ln (a^2 \sin^2 x + 6 \cos^2 x) dx.$$

$$18. \int_0^{\pi/2} \ln (9 \sin^2 x + b^2 \cos^2 x) dx.$$

$$19. \int_0^{\pi/2} \ln (\alpha^2 \sin^2 x + 10 \cos^2 x) dx.$$

$$20. \int_0^{\pi/2} \ln (8 \sin^2 x + b^2 \cos^2 x) dx.$$

$$21. \int_0^{\pi/2} \ln (a^2 \sin^2 x + 12 \cos^2 x) dx.$$

$$22. \int_0^{\pi/2} \ln (11 \sin^2 x + \beta^2 \cos^2 x) dx.$$