Canonical f-structures of Kill f class on homogeneous Φ-spaces

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Let's consider **Kill f** (Killing f-structures) [1] of *the generalized Hermitian geometry* (see, for example, [2]). The defining property for Killing f-structures is

$$\nabla_{fX}(f)fX = 0,$$

where f is a *metric f-structure* on a (pseudo)Riemannian manifold (M, g), ∇ is the Levi-Civita connection of (M, g), $X, Y \in \mathfrak{X}(M)$.

The motivation of new theorems below is [3], [4] where canonical base f-structure on homogeneous Φ-spaces of order $k = 4, 5$ are investigated against **Kill f** class in the case of naturally reductive metric. So, analyzing defining property $\nabla_{fX}(f)fX = 0$, on homogeneous Φ-spaces of order $k = 4, 5, 6$ for set of "diagonal" metrics (see [5] for the references and propositions used in the proof) we prove theorems below.

Theorem 1. *Let $M = G/H$ be a homogeneous Φ-space of order $k = 4$ with a "diagonal" metric. The canonical base f-structure f_1 is of class* **Kill f** *if and only if $\frac{\lambda_1}{\lambda_2} = \frac{3}{4}$ or $[\mathfrak{m}_2, \mathfrak{m}_1] = 0$.*

Theorem 2. *Let $M = G/H$ be a homogeneous Φ-space of order $k = 5$ with a "diagonal" metric. The canonical base f-structure f_1 is of class* **Kill f** *if and only if both:*

a) $\frac{\lambda_1}{\lambda_2} = \frac{3}{4}$ *or* $[\mathfrak{m}_2, \mathfrak{m}_1] = 0$. b) $[\mathfrak{m}_2, \mathfrak{m}_1] \subset \mathfrak{m}_1$.

The canonical base f-structure f_2 is of class **Kill f** *if and only if both:*

a) $\frac{\lambda_2}{\lambda_1} = \frac{3}{4}$ *or* $[\mathfrak{m}_2, \mathfrak{m}_1] = 0$. b) $[\mathfrak{m}_2, \mathfrak{m}_1] \subset \mathfrak{m}_2$.

Theorem 3. *Let $M = G/H$ be a homogeneous Φ-space of order $k = 6$ with "diagonal" metric. The canonical base f-structure f_1 is of class* **Kill f** *if and only if the following conditions simultaneously hold:*

a) $\lambda_1 + \lambda_2 - \lambda_3 = 0$ *or* $[\mathfrak{m}_3, \mathfrak{m}_1] = 0$. b) $\lambda_2 = \lambda_3$ *or* $[\mathfrak{m}_3, \mathfrak{m}_2] = 0$.

c) $\frac{\lambda_1}{\lambda_2} = \frac{3}{4}$ *or* $[\mathfrak{m}_2, \mathfrak{m}_1] \subset \mathfrak{m}_3$. d) $\lambda_3 + \lambda_1 - \lambda_2 = 0$ *or* $[\mathfrak{m}_2, \mathfrak{m}_1] \subset \mathfrak{m}_1$.

Theorem 4. *Let $M = G/H$ be a homogeneous Φ-space of order $k = 6$ with "diagonal" metric. The canonical base f-structure f_2 is of class* **Kill f** *if and only if the following conditions simultaneously hold:*

a) $\lambda_1 + \lambda_2 - \lambda_3 = 0$ *or* $[\mathfrak{m}_3, \mathfrak{m}_2] = 0$. b) $\lambda_1 = \lambda_3$ *or* $[\mathfrak{m}_3, \mathfrak{m}_1] = 0$.

c) $\lambda_3 + \lambda_2 - \lambda_1 = 0$ *or* $[\mathfrak{m}_2, \mathfrak{m}_1] = 0$. d) $[\mathfrak{m}_2, \mathfrak{m}_1] \subset \mathfrak{m}_3$.

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References

1. A. S. Gritsans. *On the geometry of Killing f-manifolds* // Russian Mathematical Surveys, 1990. Vol. 45, no. 4. P. 168–169.

2. V. F. Kirichenko. *Quasi–homogeneous manifolds and generalized almost Hermitian structures* // Math. USSR, Izv., 1984. Vol. 23, no. 3. P. 473–486.

3. V. V. Balashchenko. *Naturally reductive Killing f-manifolds* // Uspekhi Mat. Nauk, 1999. Vol. 54, no. 3. P. 151–152.

4. V.V. Balashchenko, Yu.G. Nikonorov, E.D. Rodionov, V.V. Slavsky. *Homogeneous spaces: theory and applications: monograph.* Hanty-Mansijsk, Polygrafist, 2008 (in Russian).

5. A. S. Samsonov. *Nearly Kähler and Hermitian f-structures on homogeneous Φ-spaces of order k with the special metrics* // Sib. Math. Journal, 2011. Vol. 52, no. 6. P. 904–915.