Advances in Algebra and Applications

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Belarusian State University, 4 Nezavisimosti avenue, Room 120

Ivan Arzhantsev. Gorenstein local algebras and additive actions on projective hypersurfaces

Let us say that an additive action on an algebraic variety X is an effective regular action with an open orbit on X of a commutative unipotent linear algebraic group. In 1999, Hassett and Tschinkel established a correspondence between artinian local commutative associative algebras with unit and additive actions on projective spaces. It turns out that this approach may be applied to the study of additive actions on other projective varieties. In this talk we discuss additive actions on projective hypersurfaces. The case of non-degenerate hypersurfaces corresponds to Gorenstein local algebras. We plan to present several results on existence and uniqueness of additive actions. The talk is based on a joint work with Julia Zaitseva.

Jean-Philippe Furter. Borel subgroups of the Cremona group

Following Demazure and Serre, the Cremona group is endowed with a natural topology. Borel subgroups are then defined as the maximal closed connected solvable subgroups. In this talk, we describe all Borel subgroups of the complex plane Cremona group. It turns out that their rank may be equal to 0,1 or 2 (the rank of a closed subgroup of the Cremona group being defined as the maximal dimension of a torus contained in it). In principle, this fact answers a question of Popov. We also show that all Borel subgroups of rank 1 or 2 are conjugate, but that this result no longer holds for Borel subgroups of rank 0. This is a joint work with I. Hedén.

Sergey Gorchinskiy. Milnor K-groups of nilpotent extensions

The talk is based on a series of common works with Dimitrii Tyurin and with Denis Osipov. We prove a version of the famous Goodwillie's theorem with algebraic K-groups being replaced by Milnor K-groups. Namely, given a commutative ring R with a nilpotent ideal I, $I^N = 0$, such that the quotient R/I splits, we study relative Milnor K-groups $K^M_{n+1}(R, I)$, $n \geqslant 0$. Provided that the ring R has enough invertible elements in a sense, these groups are related to the quotient of the module of relative differential forms $\Omega^n_{R,I}/d\,\Omega^{n-1}_{R,I}$. This holds in two different cases: when $N!$ is invertible in R and when R is a complete p-adic ring with a lift of Frobenius. However, the approaches and constructions are different in these cases.

Maria Grechkoseeva. Recognition of finite groups by the set of element orders and related problems

A finite group G is said to be recognizable by the set of element orders if G is uniquely, up to isomorphism, defined by this set in the class of finite groups. For example, the alternating group of degree 5 is recognizable since it is the unique finite group whose set of element orders is equal to $\{1, 2, 3, 5\}$. More generally, the problem of recognition by the set of element orders is said to solved for a group G if we know the number of different finite groups having the same set of element orders as G. The goal of the talk is to survey recent results concerning this recognition problem, with particular emphasis given to recognition of finite simple groups. Also we discuss the related problem of finding the element orders of finite almost simple groups.

Valentina Kiritchenko. Khovanskii–Pukhlikov rings and divided difference operators

Schubert calculus was originally developed to answer purely geometric questions. For instance, how many lines intersect 4 given lines in a 3-space? However, main tools of Schubert calculus such as cohomology rings of flag varieties and push-pull operators can be defined and studied in purely algebraic terms. In the last decade, an algebraic side of Schubert calculus was partially interpreted in convex geometric terms (in the spirit of the theory of Newton polytopes). I will talk about algebraic methods and results of Schubert calculus from this convex geometric perspective. All necessary definitions will be given in the talk.

Ivan Loseu. Quantization and representations

Quantization is a hypothetical passage from a classical mechanical system to a quantum one. Algebraically, this means deforming a Poisson algebra to an associative algebra. In this talk, I will explain how recent progress on (algebraic) quantizations sheds some light on understanding certain unitary representations of complex semisimple Lie groups.

Valery Lunts. Algebraicity of vector fields over $\mathbb{C}$ and over $\mathbb{F}_p$

The notion of quasielliptic rings appeared as a result of an attempt to classify a wide class of commutative rings of operators found in the theory of integrable systems, such as rings of commuting differential, difference, differential-difference, etc. operators. They are contained in a certain noncommutative "universal" ring — a purely algebraic analogue of the ring of pseudodifferential operators on a manifold, and admit (under certain mild restrictions) a convenient algebraic-geometric description. An important algebraic part of this description is the Schur–Sato theory — a generalisation of the well known theory for ordinary differential operators. I'll talk about this theory in dimension n and about some of its unexpected applications related to the generalized Birkhoff decomposition and to the Abhyankar formula.

Andrey Mamontov. On periodic groups with a given set of element orders

The talk is dedicated to the results on the local finiteness of periodic groups with a given set of element orders. In particular it includes the results establishing recognizability of some non-abelian finite simple groups by spectrum in the class of all groups.

Victor Mazurov. Periodic groups saturated with finite simple groups

Let M be a set of groups. By definition, a group G is saturated with groups from M (or is saturated with M) if every finite subgroup of G is contained in a subgroup which is isomorphic to some element of M. The talk is devoted to the problem of classification of periodic groups which are saturated with finite non-abelian simple groups.

Denis Osipov. Formal Bott–Thurston cocycle and part of formal Riemann–Roch theorem

The formal Bott–Thurston cocycle is a 2-cocycle on the group of continuous automorphisms of the ring of Laurent series over a ring with values in the group of invertible elements of this ring, where we consider the natural topology on the ring of Laurent series. This cocycle is a formal analog of the Bott–Thurston 2-cocycle on the group of orientation-preserving diffeomorphisms of the circle. We prove that the central extension given by the formal Bott–Thurston cocycle is equivalent to the 12-fold Baer sum of the determinantal central extension when the basic ring contains the field of rational numbers. As a consequence of this result we prove the part of new formal Riemann–Roch theorem for a ringed space over a scheme, where this ringed space is locally isomorphic to the sheaf of rings of Laurent series over the structure sheaf of this scheme.

Ivan Panin. On Grothendieck–Serre conjecture in mixed characteristic

It will be sketched a positive solution of the conjecture for the case of the special linear group corresponding to an Azumaya algebra over an unramified local regular ring R of mixed characteristic.

Particularly, it will be proved that for any units a, b, c in R the equation $X^2 - aY^2 - bZ^2 + abT^2 = c$ has a solution over the fraction field of R if and only if it has a solution over R.

Taras Panov. Polyhedral products, loop homology, and right-angled Coxeter groups

Using results on the topology of polyhedral products, we link distinct concepts of homotopy theory and geometric group theory. On the homotopical side, we describe the Pontryagin algebra (loop homology) of the moment-angle complex $\mathcal{Z}_K$. On the group-theoretical side, we describe the structure of the commutator subgroup RC'_K of a right-angled Coxeter group RC_K, viewed as the fundamental group of the real moment-angle complex $\mathcal{R}_K$. For a flag simplicial complex K, we present a minimal generator set for the Pontryagin algebra $H_*(\Omega\mathcal{Z}_K)$ and for the commutator subgroup RC'_K, and specify a necessary and sufficient combinatorial condition for $H_*(\Omega\mathcal{Z}_K)$ and RC'_K to be a free or one-relator algebra (group). We also give homological characterisations of these properties. For RC'_K, this is given by a condition on the homology group $H_2(\mathcal{R}_K)$, whereas for $H_*(\Omega\mathcal{Z}_K)$ this can be stated using the bigrading of the homology of $\mathcal{Z}_K$.

Parts of this talk are joint works with Jelena Grbic, Marina Ilyasova, George Simmons, Stephen Theriault, Yakov Veryovkin and Jie Wu.

Viktor Petrov. Isotropy of Tits construction

Tits construction produces a Lie algebra out of a composition algebra and an exceptional Jordan algebra. The type of the result is F_4, ${}^2E_6$, E_7 or E_8 when the composition algebra has dimension 1,2,4 or 8 respectively. Garibaldi and Petersson noted that the Tits index ${}^2E_6^{35}$ cannot occur as a result of Tits construction. Recently Alex Henke proved that the Tits index E_7^{66} is also not possible. We push the analogy further and show that Lie algebras of Tits index E_8^{133} don't lie in the image of the Tits construction. The proof relies on basic facts about symmetric spaces and our joint result with Garibaldi and Semenov about isotropy of groups of type E_7 in terms of the Rost invariant. This is a part of a work joint with Simon Rigby.

Alena Pirutka. Recent progress on rationality

Recall that an algebraic variety is rational if it is birational to a projective space. In this talk I will review classical results for curves, surfaces, and threefolds, and then I will report on progress in the last decade. This includes rationality of hypersurfaces, and deformation properties.

Vladimir Popov. Realizing Root Systems, their Weyl Groups, and Root Lattices in Number Fields

As is known, the set of all algebraic integers with norm 1 and 3 in a cyclotomic field generated by a cubic root of unity, is the root system of type G_2. The talk is aimed to discuss the possibility of realizing root systems, their Weyl groups, and root lattices in the form of objects naturally associated with number fields.

Yuri Prokhorov. Singular Del Pezzo varieties

A del Pezzo variety X is a Fano variety whose anticanonical class has the form $-K_X = (n-1)A$, where A is an ample line bundle and n is the dimension of X. This is a higher dimensional analog of the notion of del Pezzo surfaces. I am going to discuss biregular and birational classifications of del Pezzo varieties admitting terminal singularities.

The talk is based on a joint work with Alexander Kuznetsov.

Victor Przyjalkowski. Hodge level of weighted complete intersections

Hodge level, first introduced by Rappoport in 1972, is the maximal width of Hodge diamond of an algebraic variety. Roughly speaking, it measures how complicated the variety can be from homological point of view. The well known method going back to Griffiths enables one to compute Hodge numbers for weighted complete intersections. Using it we classify all smooth Fano weighted complete intersections with small Hodge level. It turns out that Hodge levels of all of these varieties are small by categorical reasons. On the contrary, we expect that Hodge levels of small weighted complete intersections of general type are maximally possible. We confirm this expectation in some cases. We also show that this maximality may fail for the quasi-smooth case.

Andrei Rapinchuk. Linear algebraic groups with good reduction and the genus problem

We will first discuss the notion of good reduction with respect to a discrete valuation for a reductive linear algebraic group, and then formulate a finiteness conjecture for the forms having good reduction at a divisorial set of places of a finitely generated field. This conjecture provides a uniform approach to several problems including the genus problem for division algebras and algebraic groups, the properness of the global-to-local map in Galois cohomology and the analysis of weakly commensurable Zariski-dense subgroups — the latter also has some geometric applications. We will present some of the available results on the finiteness conjecture — see the survey article A.R., I. Rapinchuk, "Linear algebraic groups with good reduction", Res. Math. Sci. 7(2020) for more information.

Constantin Shramov. Conic bundles

Consider a conic bundle over a smooth incomplete curve C, i.e. a smooth surface S with a proper surjective morphism to C such that the push-forward of the structure sheaf of S coincides with the structure sheaf of C, and the anticanonical class of S is ample over C. I will tell about a necessary and sufficient condition for the existence of an extension of this conic bundle to the completion of C. The talk is based on a joint work in progress with V. Vologodsky.

Irina Suprunenko. On the behaviour of unipotent elements from subsystem subgroups of small ranks in irreducible representations of the classical algebraic groups in positive characteristic

The behaviour of unipotent elements from subsystem subgroups of small ranks in modular irreducible representations of the classical algebraic groups is investigated. In the talk, the principal attention will be given to regular unipotent elements from subsystem subgroups of type A_5 and C_3 in representations of groups of type A_n and C_n, respectively. For infinitesimally irreducible representations, it is proved that the images of such elements have Jordan blocks of all a priori possible sizes if some 10 consecutive coefficients of the highest weight in the first case and last 6 consecutive coefficients of this weight in the second one satisfy certain special conditions. These are joint results with T. S. Busel.

Anastasia Stavrova. Non-stable K_1-functors and R-equivalence on reductive groups

Let A be a commutative ring. The elementary subgroup $E_n(A)$ of $SL_n(A)$ is the subgroup generated by the elementary transvections $e + te_{ij}$, where $1 \leqslant i, j \leqslant n$ are distinct and t is any element of A. This notion generalizes to any reductive A-group scheme G satisfying a suitable isotropy condition. Namely, one defines the elementary subgroup $E(A)$ of the group of A-points $G(A)$ as the subgroup generated by the A-points of unipotent radicals of parabolic subgroups of G. The functor $K_1^G(-) = G(-)/E(-)$ on the category of commutative A-algebras is called the non-stable K_1-functor associated to G. If $A = k$ is a field and G is semisimple, $E(k)$ is nothing but the group $G(k)^+$ introduced by J. Tits; in this case $K_1^G(k) = W(k, G)$ is also called the Whitehead group of G, and its computation is the subject of the Kneser–Tits problem. In this context, it has been known for some time that if G is simply connected, then $K_1^G(k)$ coincides with the R-equivalence class group $G(k)/R$ in the sense of Yu. Manin. We generalize this identification to reductive groups over rings other than fields and apply it to the study of birational properties of K_1^G and $G(-)/R$. The talk is based on a joint work with P. Gille.

Dmitri Timashev. Galois cohomology of real algebraic groups

I shall speak about joint results with Mikhail Borovoi on computation of Galois cohomology of linear algebraic groups over real numbers. The real case is crucial in computation of Galois cohomology over number fields. The Levi decomposition reduces the problem to the case of reductive groups and a theorem of Borovoi (1988) reduces computation to maximal anisotropic tori. Based on this, we obtain an explicit combinatorial description of Galois cohomology for a real reductive group in terms of some special integer labelings of its affine Dynkin diagram and some subquotients of the cocharacter lattice of the central torus. As a by-product, we obtain a transparent description for the component group of the real locus of a connected reductive group in terms of the cocharacter lattice of a maximal split torus, which reinforces a classical result of Matsumoto (1964).

Yuri Tschinkel. Equivariant birational geometry and algebraic tori

I will discuss the definition of new invariants in equivariant birational geometry, as well as their properties and applications. The talk is based on a joint work with A. Kresch.

Nikolai Vavilov. Bounded generation of Chevalley groups, and around

We state several results on bounded elementary generation and bounded commutator width for Chevalley groups over Dedekind rings of arithmetic type in positive characteristic. In particular, Chevalley groups of rank ≥ 2 over polynomial rings $\mathbb{F}_q[t]$ and Chevalley groups of rank ≥ 1 over Laurent polynomial $\mathbb{F}_q[t, t^{-1}]$ rings, where $\mathbb{F}_q$ is a finite field of q elements, are boundedly elementarily generated. We sketch several proofs, and establish rather plausible explicit bounds, which are better than the known ones even in the number case. Using these bounds we can also produce sharp bounds of the commutator width of these groups. We also mention several applications (such as Kac–Moody groups and first order rigidity) and possible generalisations (verbal width, strong bounded generation, etc.)

The talk is based on a joint work with Boris Kunyavskii, Eugene Plotkin.

Maxim Vsemirnov. TBA

Vyacheslav Yanchevskii. Henselian division algebras and reduced unitary Whitehead groups for outer forms of anisotropic algebraic groups of type A_n

Results on the structure of involutory Henselian weakly ramified division algebras that are used for finding formulas for computing reduced unitary Whitehead groups of outer forms of anisotropic algebraic groups of type A_n, are obtained.

Alexander Zheglov. The Schur–Sato theory for quasi-elliptic rings and some of its applications

The notion of quasielliptic rings appeared as a result of an attempt to classify a wide class of commutative rings of operators found in the theory of integrable systems, such as rings of commuting differential, difference, differential-difference, etc. operators. They are contained in a certain noncommutative "universe" ring — a purely algebraic analogue of the ring of pseudodifferential operators on a manifold, and admit (under certain mild restrictions) a convenient algebraic-geometric description. An important algebraic part of this description is the Schur–Sato theory — a generalisation of the well known theory for ordinary differential operators. I'll talk about this theory in dimension n and about some of its unexpected applications related to the generalized Birkhoff decomposition and to the Abhyankar formula.