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ON THE RELEVANCE OF FRACTIONAL INTEGRO-DIFFERENTIATION IN THE PREPARATION OF MASTERS OF MATHEMATICAL SPECIALTIES

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The problem of expediency of inclusion in the training of masters of mathematical specialties of the section on fractional integro-differentiation, due to the high applied demand of this section and the lack of development of the method of its inclusion in training, is considered. Illustrations of applied tasks as actual aspects of using fractional calculus, establishing analogies between its properties and properties of differentiation of entire orders, identifying the relationship of the analytical apparatus and numerical methods are important didactic factors of motivation and content appeal of this section for masters of mathematical specialties.

Keywords: integer and fractional integro-differentiation, masters training, mathematical specialties, heat conduction equation.

1. Introduction

In the context of the transition of higher education to a two-stage bachelor-master education, the task of developing master's degree programs becomes particularly relevant. With regard to mathematical specialties, the development of such special courses is of particular interest, which provide for the solution of a triune problem: on the one hand, the development of new mathematical methods, on the other – applied aspects of the use of the studied mathematical apparatus, on the third – the development and development of computer modeling methods.

One of the options for expanding and deepening the content of mathematical training of mathematics students in the magistracy is the study of materials on fractional integro-differentiation. The relevance of including such a section in the training program for masters of mathematical specialties is due to the fact that the main purpose of attracting mathematical apparatus to solving applied problems is to build and study the properties of models that are most adequate to real processes, taking into account the given initial conditions. The more precisely it is necessary to investigate the real process, the more "sensitive" mathematical apparatus should be used [1]. …

2. Examples of using the fractional integro-differentiation apparatus

To describe, for example, the processes of heat propagation in highly porous media, a modified Fick's law is used, which requires the involvement of the mathematical apparatus of fractional integro-differential calculus. Fractional derivatives are introduced into the classical equation of thermal conductivity both in space and in time. Initial boundary value problems arise for differential equations with fractional derivatives [2]. Here is an example of such a task.

Formulation of the initial boundary value problem on the example of the heat equation. Comparison of methods based on different definitions of fractional derivatives was carried out on test problems of thermal conductivity, then tested on more complex problems with known solutions [3].

Consider the formulation of the problem in the simplest case:

$$D_t^{\gamma} u(x,t) = C D_x^{\alpha} u(x,t), \quad 0 < \gamma \leq 2,\ 1 \leq \alpha \leq 2$$

with initial conditions

$$u(x,0) = u_0(x)$$
$$0 < \gamma \leq 2,\ 1 \leq \alpha \leq 2 \quad \dots$$

3. Analogies with ordinary differentiation

Let's consider some analogies: integro-differential operators of non-integer orders are linear operators, as well as their integer counterparts. For differentiation operations with an integer order, as is known, the formula is valid:

$$\frac{d^m}{dt^m}\left(\sum_{k=1}^{n} b_k f_k(t)\right) = \sum_{k=1}^{n} b_k \frac{d^m}{dt^m}(f_k(t)) \tag{1.1}$$

where m is an integer. For derivatives of fractional order we have a similar relation (1.2), …

Formulas (1.1 – 1.4) combine two properties of linearity of differentiation and integration operators: the constant (b_k) can be taken as the sign of differentiation (integration), and the derivative (integral) of the sum of functions is equal to the sum of derivatives (integrals) of functions of the same order [7, p.433]. …

4. Conclusion

Despite some difficulty in introducing this new device for students, it is advisable to study it for students of mathematical specialties, since it allows solving applied problems of automatic control, problems of thermal conductivity (diffusion) in fractal media, calculation of thermodynamic properties of the surface, solving equations of thermodynamics of material media, problems of continuum mechanics, and is also intensively used in the construction of mathematical models different processes in real environments.

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